International Journal of Mathematics and Computer Application Research (IJMCAR)
ISSN (P): 2250-2378; ISSN (E): 2319-4510
Vol. 9, Issue 2, Dec 2019, 73–82

INTUITIONISTIC FUZZY SUBSEMIRING

B. ANITHA

Annamalai University, Annamalainagar,Tamilnadu, India.

ABSTRACT:

In this paper, we introduce $\overline{Q}$-intuitionistic fuzzy set, $\overline{Q}$-intuitionistic fuzzy subsemiring. Also we study their properties.

KEYWORDS: $\overline{Q}$-intuitionistic fuzzy set & $\overline{Q}$-intuitionistic fuzzy subsemiring

Received: Jun 13, 2019; **Accepted**: Jul 03, 2019; **Published**: Oct 26, 2019; **Paper Id.**: IJMCARDEC20197

1. INTRODUCTION

Zadeh [13] in 1965 introduced fuzzy sets after which several researchers explored on the generalizations of the notion of fuzzy sets and its application to many mathematical branches. Atanassov introduced intuitionistic fuzzy set which constitute a genralization of the notion of fuzzy sets [1, 2]. A Solairaju and R. Nagarajan [8, 9, 10] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. S. Hemalatha, et.al. [3] introduced the concept of Q-fuzzy subring of a ring and established some results. A study on anti Q-fuzzy subsemiring of a semiring has been introduced by Vanathi, et.al. [11]. Some theorems in Q-intuitionistic fuzzy subsemiring of a semiring has been introduced by Vanathi et.al. [12]. O. Ratnabala Devi [7] introduced the concept of intuitionistic Q-fuzzy ideals of Near-rings. In this paper we introduce the concept of $\overline{Q}$-intuitionistic fuzzy subset, $\overline{Q}$-intuitionistic fuzzy subsemiring. So far all Q −fuzzy subsets of rings, semirings, near-rings are studied with Q as a set only. In this paper we introduce the concept of $\overline{Q}$-intuitionistic fuzzy subset of a semiring where $(Q,.)$ is a semigroup.

2. PRELIMINARIES

Definition 2.1 *Let X be a non empty set and Q be a non empty set. A Q-fuzzy subset A of X is a function $A: X \times Q \to$* $[0,1]$.

Definition 2.2 *Let $(R,+,\cdot)$ be a semiring. A Q-fuzzy subset A of R is said to be a Q-fuzzy subsemiring of R if it satisfies the following conditions: [(i)]*

- $.\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$, for all $x, y \in R$

Definition 2.3 *A Q-intuitionistic fuzzy subset A in X is defined as an object of the form $A = \{\langle (x,q), \mu_A(x,q), \nu_A(x,q)\rangle /$ $x \in X$ and $q \in Q\}$, where $\mu_A: X \times Q \to [0,1]$ and $\nu_A: X \times Q \to [0,1]$ define the degree of membership and the degree of non-membership of the element x in X and q in Q respectively and for every x in X and q in Q satisfying* $0 \leq \mu_A(x,q) + \nu_A(x,q) \leq 1$.

Definition 2.4 *Let $(R,+,\cdot)$ be a semiring. A Q-intuitionistic fuzzy subset A of R is said to be a Q-intuitionistic fuzzy subsemiring of R if it satisfies the following conditions: [(i)]*

- $\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\nu_A(x+y,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\}$
- $\nu_A(xy,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\}, \forall x,y \in R$ and $q \in Q$.

3. $\overline{Q}$-INTUITIONISTIC FUZZY SUBSEMIRING

Definition 3.1 *Let* $(R,+,\cdot)$ *be a semiring. Let* $(Q,\cdot)$ *be a semigroup. A map* $A: R \times Q \to [0,1]$ *is said to be a* $\overline{Q}$*-fuzzy subset of* R.

Definition 3.2 *Let* $(R,+,,\cdot)$ *be a semiring. Let* $(Q,\cdot)$ *be a semigroup. An* $\overline{Q}$*-intuitionistic fuzzy set is defined as* $A = \{<(x,q), \mu_A(x,q), \nu_A(x,q)>/x \in R, q \in Q\}$ *where the function* $\mu_A: R \times Q \to [0,1]$ *and* $\nu_A: R \times Q \to [0,1]$ *denote the degree of membership and degree of non-membership for each element* $x \in R, q \in Q$ *to the set* A *respectively and* $0 \leq \mu_A(x,q) + \nu_A(x,q) \leq 1$, *for each* $x \in R, q \in Q$.

Definition 3.3 *Let* $(R,+,\cdot)$ *be a semiring. A* $\overline{Q}$*-fuzzy subset* A *of* R *is said to be a* $\overline{Q}$*-fuzzy subsemiring of* R *if it satisfies the following conditions:*

- $\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(x,q,q_2) \geq \min\{\mu_A(x,q_1), \mu_A(x,q_2)\}, \forall x,y \in R$ and q, q_1, q_2 in Q.

Definition 3.4 *Let* $(R,+,\cdot)$ *be a semiring. A* $\overline{Q}$*-fuzzy subset* A *of* R *is said to be an anti* $\overline{Q}$*-fuzzy subsemiring of* R *if it satisfies the following conditions:*

- $\mu_A(x+y,q) \leq \max\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(xy,q) \leq \max\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(x,q,q_2) \leq \max\{\mu_A(x,q_1), \mu_A(x,q_2)\}$

Definition 3.5 *Let* $(R,+,\cdot)$ *be a semiring. A* $\overline{Q}$*-fuzzy subset* A *of* R *is said to be a* $\overline{Q}$*-intuitionistic fuzzy subsemiring of* R *if it satisfies the following conditions:*

- $\mu_A(x+y,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(xy,q) \geq \min\{\mu_A(x,q), \mu_A(y,q)\}$
- $\mu_A(x,q.q_2) \geq \min\{\mu_A(x,q_1), \mu_A(x,q_2)\}$
- $\nu_A(x+y,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\}$
- $\nu_A(xy,q) \leq \max\{\nu_A(x,q), \nu_A(y,q)\}$
- $\nu_A(x,q.q_2) \leq \max\{\nu_A(x,q_1), \nu_A(x,q_2)\} \forall x,y \in R$ and $q, q_1, q_2 \in Q$.

4. SOME THEOREMS OF $\overline{Q}$-INTUITIONISTIC FUZZY SUBSEMIRING

Theorem 4.1 *Intersection of any two* $\overline{Q}$*-intuitionistic fuzzy subsemiring of a semiring* R *is a* $\overline{Q}$*-intuitionistic fuzzy subsemiring of* R.

Proof. Let A and B be any two $\overline{Q}$-intuitionistic fuzzy subsemirings of a semiring R and $x, y \in R$ and $q, q_1, q_2 \in$

Q.

Let $A = \{((x,q),\mu_A(x,q),\nu_A(x,q))/x \in R and q \in Q\}$ and B=$\{((x,q),\mu_B(x,q),\nu_B(x,q))/x \in R$ and $q \in Q\}$ and also $C = A \cap B = \{((x,q),\mu_C(x,q),\nu_C(x,q))/x \in R$ and $q \in Q\}$,

where $\min\{\mu_A(x,q),\mu_B(x,q)\} = \mu_C(x,q)$ and $\max\{\nu_A(x,q),\nu_B(x,q)\} = \nu_C(x,q)$.

Now, $\mu_C(x+y,q) = \min\{\mu_A(x+y,q),\mu_B(x+y,q)\}$

$$\geq \min\{\min\{\mu_A(x,q),\mu_A(y,q)\},\min\{\mu_B(x,q),\mu_B(y,q)\}\}$$

$$= \min\{\min\{\mu_A(x,q),\mu_B(y,q)\},\min\{\mu_A(x,q),\mu_B(y,q)\}\}$$

$$= \min\{\mu_C(x,q),\mu_C(y,q)\}$$

$\therefore \mu_C(x+y,q) \geq \min\{\mu_C(x,q),\mu_C(y,q)\} \forall x,y \in R$ and $q \in Q$

and $\mu_C(xy,q) = \min\{\mu_A(xy,q),\mu_B(xy,q)\}$

$$\geq \min\{\min\{\mu_A(x,q),\mu_A(y,q)\},\min\{\mu_B(x,q),\mu_B(y,q)\}\}$$

$$= \min\{\min\{\mu_A(x,q),\mu_B(y,q)\},\min\{\mu_A(y,q),\mu_B(y,q)\}\}$$

$$= \min\{\mu_C(x,q),\mu_C(y,q)\}$$

$\therefore \mu_C(xy,q) \geq \min\{\mu_C(x,q),\mu_C(y,q)\} \forall x,y \in R$ and $q \in Q$

$\mu_C(x,q_1.q_2) = \min\{\mu_A(x,q_1.q_2),\mu_B(x,q_1.q_2)\}$

$$\geq \min\{\min\{\mu_A(x,q_1),\mu_A(x,q_2)\},\min\{\mu_B(x,q_1),\mu_B(x,q_2)\}\}$$

$$= \min\{\min\{\mu_A(x,q_1),\mu_B(x,q_1)\},\min\{\mu_A(x,q_2),\mu_B(y,q_2)\}\}$$

$$= \min\{\mu_C(x,q_1),\mu_C(x,q_2)\}$$

$\therefore \mu_C(x,q.q_2) \geq \min\{\mu_C(x,q_1),\mu_C(x,q_2)\} \forall x \in R$ and $q_1,q_2 \in Q$

Now $\nu_C(x+y,q) = \max\{\nu_A(x+y,q),\nu_B(x+y,q)\}$

$$\leq \max\{\max\{\nu_A(x,q),\nu_A(y,q)\},\max\{\nu_B(x,q),\nu_B(y,q)\}\}$$

$$= \max\{\max\{\nu_A(x,q),\nu_B(x,q)\},\max\{\nu_A(y,q),\nu_B(y,q)\}\}$$

$$= \max\{\nu_C(x,q),\nu_C(y,q)\}$$

$\therefore \nu_C(x+y,q) \leq \max\{\nu_C(x,q),\nu_C(y,q)\} \forall x,y \in R$ and $q \in Q$

Now $\nu_C(xy,q) = \max\{\nu_A(xy,q),\nu_B(xy,q)\}$

$$\leq \max\{\max\{\nu_A(x,q),\nu_A(y,q)\},\max\{\nu_B(x,q),\nu_B(y,q)\}\}$$

$$= \max\{\max\{\nu_A(x,q),\nu_B(x,q)\},\max\{\nu_A(y,q),\nu_B(y,q)\}\}$$

$$= \max\{\nu_C(x,q),\nu_C(y,q)\}$$

$\therefore \nu_C(xy,q) \leq \max\{\nu_C(x,q),\nu_C(y,q)\} \forall x,y \in R$ and $q \in Q$

and $\nu_C(x,q_1.q_2) = \max\{\nu_A(x,q_1.q_2),\nu_B(x,q_1.q_2)\}$

$$\leq \max\{\max\{\nu_A(x,q_1),\nu_A(y,q_2)\},\max\{\nu_B(x,q_1),\nu_B(y,q_2)\}\}$$

$$= \max\{\max\{\nu_A(x,q_1),\nu_B(x,q_1)\},\max\{\nu_A(y,q_2),\nu_B(y,q_2)\}\}$$

$$= \max\{\nu_C(x,q_1),\nu_C(y,q_2)\}$$

$\therefore \nu_C(x,q_1.q_2) \leq \max\{\nu_C(x,q_1),\nu_C(y,q_2)\} \forall x \in R$ and $q_1,q_2 \in Q$

Hence the intersection of any two $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring R is a$\overline{Q}$-intuitionistic fuzzy subsemiring of R.

Theorem 4.2 *The intersection of a family of $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring R is a $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring R.*

Proof. Let $\{v_i; i \in I\}$ be a family of $\overline{Q}$-intuitionistic fuzzy subsemiring of a ring R and let$A = \bigcap_{i\in I} v_i$.

Let $x,y \in R$ and $q,q_1,q_2 \in Q$.

Then $\mu_A(x+y,q) = \inf\limits_{i\in I}\mu_{v_i}(x+y,q) \geq \inf\limits_{i\in I}\min\{\mu_{v_i}(x,q),\mu_{v_i}(y,q)\}$

$$= \min\{\inf_{i\in I}\mu_{v_i}(x,q),\inf_{i\in I}\mu_{v_i}(y,q)\} = \min\{\mu_A(x,q),\mu_A(y,q)\}$$

$\therefore \mu_A(x+y,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$ for all $x,y \in R$ and $q \in Q$.

Now $\mu_A(xy,q) = \inf\limits_{i\in I}\mu_{v_i}(xy,q) \geq \inf\limits_{i\in I}\min\{\mu_{v_i}(x,q),\mu_{v_i}(y,q)\}$

$$= \min\left\{\inf_{i\in I}\mu_{v_i}(x,q),\inf_{i\in I}\mu_{v_i}(y,q)\right\} = \min\{\mu_A(x,q),\mu_A(y,q)\}$$

$\therefore \mu_A(xy,q) \geq \min\{\mu_A(x,q),\mu_A(y,q)\}$ for all $x,y \in R$ and $q \in Q$.

and $\mu_A(x,q_1.q_2) = \inf\limits_{i\in I}\mu_{v_i}(x,q_1.q_2) \geq \inf\limits_{i\in I}\min\{\mu_{v_i}(x,q_1),\mu_{v_i}(x,q_2)\}$

$$= \min\{\inf_{i\in I}\mu_{v_i}(x,q_1),\inf_{i\in I}\mu_{v_i}(x,q_2)\} = \min\{\mu_A(x,q_1),\mu_A(x,q_2)\}$$

$\therefore \mu_A(x,q_1.q_2) \geq \min\{\mu_A(x,q_1),\mu_A(x,q_2)\}$ for all $x \in R$ and $q_1,q_2 \in Q$.

Now $\nu_A(x+y,q) = \sup\limits_{i\in I}\nu_{v_i}(x+y,q) \leq \sup\limits_{i\in I}\max\{\nu_{v_i}(x,q),\nu_{v_i}(y,q)\}$

$= \max\{\sup\limits_{i\in I}\nu_{v_i}(x,q),\sup\limits_{i\in I}\nu_{v_i}(y,q)\} = \max\{\nu_A(x,q),\nu_A(y,q)\} \forall x,y \in R$ and $q \in Q$.

$$\nu_A(xy,q) = \sup_{i\in I}\nu_{v_i}(xy,q) \leq \sup_{i\in I}\max\{\nu_{v_i}(x,q),\nu_{v_i}(y,q)\}$$

$= \max\{\sup\limits_{i\in I}\nu_{v_i}(x,q),\sup\limits_{i\in I}\nu_{v_i}(y,q)\} = \max\{\nu_A(x,q),\nu_A(y,q)\} \forall x,y \in R$ and $q \in Q$.

$$\nu_A(x,q_1.q_2) = \sup_{i\in I}\nu_{v_i}(x,q_1.q_2) \geq \sup_{i\in I}\max\{\nu_{v_i}(x,q_1),\nu_{v_i}(y,q_2)\}$$

$= \max\{\sup\limits_{i\in I}\nu_{v_i}(x,q_1),\sup\limits_{i\in I}\nu_{v_i}(y,q_2)\} = \max\{\nu_A(x,q_1),\nu_A(y,q_2)\} \forall x \in R$ and $q_1,q_2 \in Q$.

That is A is a $\overline{Q}$-intuitionistic fuzzy subsemiring of a ring R. Hence the intersection of a family of $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring R is a $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring R.

Theorem 4.3 *If A and B are any two $\overline{Q}$-intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively, then $A \times B$ is a $\overline{Q}$-intuitionistic fuzzy subsemiring of $R_1 \times R_2$.*

Proof. Let A and B be two $\overline{Q}$-intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively.

Let $x_1, x_2 \in R_1, y_1, y_2 \in R_2$. Then (x_1, y_1) and (x_2, y_2) are in $R_1 \times R_2$.

Now $\mu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] = \mu_{A\times B}((x_1 + x_2, y_1 + y_2), q)$

$$= \min\{\mu_A(x_1 + x_2, q), \mu_B(y_1 + y_2, q)\}$$

$$\geq \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\}, \mu_B(y_1, q), \mu_B(y_2, q)\}\}$$

$$= \min\{\min\{\mu_A(x, q), \mu_B(y_1, q)\}, \min\{\mu_A(x_2, q), \mu_B(y_2, q)\}\}$$

$$= \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$$

$\therefore \mu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] \geq \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$

Also $\mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] = \mu_{A\times B}((x_1x_2, y_1y_2), q)$

$$= \min\{\mu_A(x_1x_2, q), \mu_B(y_1y_2, q)\}$$

$$\geq \min\{\min\{\mu_A(x_1, q), \mu_A(x_2, q)\}, \min\}, \mu_B(y_1, q), \mu_B(y_2, q)\}\}$$

$$= \min\{\min\{\mu_A(x, q), \mu_B(y_1, q)\}, \min\{\mu_A(x_2, q), \mu_B(y_2, q)\}\}$$

$$= \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$$

$\therefore \mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] \geq \min\{\mu_{A\times B}((x_1, y_1), q), \mu_{A\times B}((x_2, y_2), q)\}$

Also $\mu_{A\times B}[(x_1, y_1), q_1.q_2] = \min\{\mu_A(x_1, q_1.q_2), \mu_B(y_1, q_1.q_2)\}$

$$= \min\{\mu_A(x_1 + x_2, q), \mu_B(y_1 + y_2, q)\}$$

$$\geq \min\{\min\{\mu_A(x_1, q_1), \mu_A(x_1, q_2)\}, \min\}, \mu_B(y_1, q_1), \mu_B(y_2, q_2)\}\}$$

$$= \min\{\min\{\mu_A(x_1, q_1), \mu_B(y_1, q_1)\}, \min\{\mu_A(x_2, q_2), \mu_B(y_2, q_2)\}\}$$

$$= \min\{\mu_{A\times B}((x_1, y_1), q_1), \mu_{A\times B}((x_1, y_1), q_2)\}$$

$\therefore \mu_{A\times B}[(x_1, y_1), q_1.q_2] \geq \min\{\mu_{A\times B}((x_1, y_1), q_1), \mu_{A\times B}((x_1, y_1), q_2)\}$

Now $\nu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] = \nu_{A\times B}((x_1 + x_2, y_1 + y_2), q)$

$$= \max\{\nu_A(x_1 + x_2, q), \nu_B(y_1 + y_2, q)\}$$

$$\leq \max\{\max\{\nu_A(x_1, q), \nu_A(x_2, q)\}, \max\{\nu_B(y_1, q), \nu_B(y_2, q)\}\}$$

$$= \max\{\max\{\nu_A(x_1, q), \nu_B(y_1, q)\}, \max\{\nu_A(x_2, q), \nu_B(y_2, q)\}\}$$

$$= \max\{\nu_{A\times B}((x_1, y_1), q), \nu_{A\times B}((x_2, y_2), q)\}$$

$\therefore \mu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] \leq \max\{\nu_{A\times B}((x_1, y_1), q), \nu_{A\times B}((x_2, y_2), q)\}$

Also $\nu_{A\times B}[(x_1, y_1)(x_2, y_2), q] = \nu_{A\times B}((x_1x_2, y_1y_2), q)$

$$= \max\{\nu_A(x_1x_2, q), \nu_B(y_1y_2, q)\}$$

$$\leq \max\{\max\{\nu_A(x_1,q),\nu_A(x_2,q)\},\max\{\nu_B(y_1,q),\nu_B(y_2,q)\}\}$$

$$= \max\{\max\{\nu_A(x_1,q),\nu_B(y_1,q)\},\max\{\nu_A(x_2,q),\nu_B(y_2,q)\}\}$$

$$= \max\{\nu_{A\times B}((x_1,y_1),q),\nu_{A\times B}((x_2,y_2),q)\}$$

$\therefore \mu_{A\times B}[(x_1,y_1)(x_2,y_2),q] \leq \max\{\nu_{A\times B}((x_1,y_1),q),\nu_{A\times B}((x_2,y_2),q)\}$

Now $\nu_{A\times B}[(x_1,y_1),q_1.q_2] = \max\{\nu_A(x_1,q_1.q_2),\nu_B(y_1,q_1.q_2)\}$

$$\leq \max\{\max\{\nu_A(x_1,q_1),\nu_A(x_1,q_2)\},\max\{\nu_B(y_1,q_1),\nu_B(y_1,q_2)\}\}$$

$$= \max\{\max\{\nu_A(x_1,q_1),\nu_B(y_1,q_1)\},\max\{\nu_A(x_2,q),\nu_B(y_2,q)\}\}$$

$$= \max\{\nu_{A\times B}((x_1,y_1),q_1),\nu_{A\times B}((x_1,y_1),q_2)\}$$

$\therefore \mu_{A\times B}[(x_1,y_1),q_1.q_2] \leq \max\{\nu_{A\times B}((x_1,y_1),q_1),\nu_{A\times B}((x_1,y_1),q_2)\}$

Hence $A\times B$ is a $\overline{Q}$-intuitionistic fuzzy subsemiring of $R_1\times R_2$.

Theorem 4.4 *Let A be a $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring H and f is an isomorphism from a ring R on to H. Then $A\circ f$ is a $\overline{Q}$-intuitionistic fuzzy subsemiring of R.*

Proof. Let x and y be arbitrary elements in R and $q,q_1,q_2\in Q$ and A be a $\overline{Q}$-intuitionistic fuzzy subsemiring of H. Then we have

$(\mu_A\circ f)(x+y,q) = \mu_A(f(x+y),q)$

$$= \mu_A(f(x)+f(y),q)$$

$$\geq \min\{\mu_A(f(x),q),\mu_A(f(y),q)\}$$

$$\geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$$

$\Rightarrow (\mu_A\circ f)(x+y,q) \geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$

Also $(\mu_A\circ f)(xy,q) = \mu_A(f(xy),q)$

$$= \mu_A(f(x)f(y),q)$$

$$\geq \min\{\mu_A(f(x),q),\mu_A(f(y),q)\}$$

$$\geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$$

$\Rightarrow (\mu_A\circ f)(xy,q) \geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$

and $(\mu_A\circ f)(x,q_1.q_2) = \mu_A(f(x),q_1.q_2)$

$$\geq \min\{\mu_A(f(x),q_1),\mu_A(f(x),q_2)\}$$

$$\geq \min\{(\mu_A\circ f)(x_1,q_1),(\mu_A\circ f)(x,q_2)\}$$

$\Rightarrow (\mu_A\circ f)(x,q_1.q_2) \geq \min\{(\mu_A\circ f)(x,q_1),(\mu_A\circ f)(x,q_2)\}$

Then we have $(\nu_A\circ f)(x+y,q) = \nu_A(f(x+y),q)$

$$= \nu_A(f(x)+f(y),q)$$

$$\leq \max\{\nu_A(f(x),q),\nu_A(f(y),q)\}$$

$$\leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$$

$\Rightarrow (\nu_A\circ f)(x+y,q) \leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$

Also $(\nu_A\circ f)(xy,q) = \nu_A(f(xy),q)$

$$= \nu_A(f(x)f(y),q)$$

$$\leq \max\{\nu_A(f(x),q),\nu_A(f(y),q)\}$$

$$\leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$$

$\Rightarrow (\nu_A\circ f)(xy,q) \leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$

and $(\nu_A\circ f)(x,q_1.q_2) = \nu_A(f(x),q_1.q_2)$

$$\leq \max\{\nu_A(f(x),q_1),\nu_A(f(x),q_2)\}$$

$$\leq \max\{(\nu_A\circ f)(x,q_1),(\nu_A\circ f)(x,q_2)\}$$

$\Rightarrow (\nu_A\circ f)(x,q_1.q_2) \leq \max\{(\nu_A\circ f)(x,q_1),(\nu_A\circ f)(x,q_2)\}$

$\therefore A\circ f$ is a $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring R.

Theorem 4.5 *Let A be a $\overline{Q}$-intuitionistic fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then $A\circ f$ is a $\overline{Q}$-intuitionistic fuzzy subsemiring of R.*

Proof. Let x and y be arbitrary elements in R and $q,q_1,q_2 \in Q$ and A be a $\overline{Q}$-intuitionistic fuzzy subsemiring of H. Then we have

$(\mu_A\circ f)(x+y,q) = \mu_A(f(x+y),q)$

$$= \mu_A(f(x)+f(y),q)$$

$$\geq \min\{\mu_A(f(x),q),\mu_A(f(y),q)\}$$

$$\geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$$

$\Rightarrow (\mu_A\circ f)(x+y,q) \geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$

Also $(\mu_A\circ f)(xy,q) = \mu_A(f(xy),q)$

$$= \mu_A(f(y)f(x),q)$$

$$\geq \min\{\mu_A(f(y),q),\mu_A(f(x),q)\}$$

$$= \min\{\mu_(f(x),q),\mu_A(f(y),q)\}$$

$$\geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$$

$\Rightarrow (\mu_A\circ f)(xy,q) \geq \min\{(\mu_A\circ f)(x,q),(\mu_A\circ f)(y,q)\}$

and $(\mu_A\circ f)(x,q_1.q_2) = \mu_A(f(x),q_1.q_2)$

$$\geq \min\{\mu_A(f(x),q_1),\mu_A(f(x),q_2)\}$$

$$\geq \min\{(\mu_A\circ f)(x,q_1),(\mu_A\circ f)(x,q_2)\}$$

$$\Rightarrow (\mu_A\circ f)(x,q_1.q_2)\geq \min\{(\mu_A\circ f)(x,q_1),(\mu_A\circ f)(x,q_2)\}$$

Now $(\nu_A\circ f)(x+y,q)=\nu_A(f(x+y),q)$

$$=\nu_A(f(x)+f(y),q)$$

$$\leq \max\{\nu_A(f(x),q),\nu_A(f(y),q)\}$$

$$\leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$$

$$\Rightarrow (\nu_A\circ f)(x+y,q)\leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$$

Also $(\nu_A\circ f)(xy,q)=\nu_A(f(xy),q)$

$$=\nu_A(f(y)f(x),q)$$

$$\leq \max\{\nu_A(f(y),q),\nu_A(f(x),q)\}$$

$$=\max\{\nu_A(f(x),q),\nu_A(f(y),q)\}$$

$$\leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$$

$$\Rightarrow (\nu_A\circ f)(xy,q)\leq \max\{(\nu_A\circ f)(x,q),(\nu_A\circ f)(y,q)\}$$

and $(\nu_A\circ f)(x,q_1.q_2)=\nu_A(f(x),q_1.q_2)$

$$\leq \max\{\nu_A(f(x),q_1),\nu_A(f(x),q_2)\}$$

$$\leq \max\{(\nu_A\circ f)(x,q_1),(\nu_A\circ f)(x,q_2)\}$$

$$\Rightarrow (\nu_A\circ f)(x,q_1.q_2)\leq \max\{(\nu_A\circ f)(x,q_1),(\nu_A\circ f)(x,q_2)\}$$

$\therefore A\circ f$ is a $\overline{Q}$-intuitionistic fuzzy subsemiring of a ring R.

REFERENCES

1. *K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems,* ***20*** *(1986) 87–96.*
2. *K.T. Atanassov, Intuitionistic fuzzy sets, Intuitionistic fuzzy sets theory and applications, Bulgaria: Physica-Verlag, A Springer-Verlag Company, (1999).*
3. *S. Hemalatha, S. Nagaranathan and K. Arjunan, Some theorem on Q-fuzzy subring of a ring, International Journal of Emerging Technology and Advanced Engineering,* ***2(12)****(2012).*
4. *K.H. Kim, On intuitionistic Q-fuzzy semiprime ideals in semigroups, Advances in Fuzzy Mathematics,* ***1(1)*** *(2006) 15–21.*
5. *S. Lekkoksung, On intuitionistic Q-fuzzy K-ideals of semirings, International Journal of Contemp. Math. Sciences,* ***7(8)*** *(2012) 389–393.*
6. *Rajesh Kumar, Fuzzy Algebra, University of Delhi Publications Division,* ***1*** *(1993).*
7. *O. Ratnabala Devi, On the intuitionistic Q-fuzzy ideals of near-rings, NIFS,* ***15(3)****(2009) 25–32.*
8. *A. Solairaju and R. Nagarajan, A new structure and construction of Q-fuzzy groups, Advances in Fuzzy Mathematics,* ***4(1)***

(2009) 23–29.

9. *A. Solairaju and R. Nagarajan, Lattice valued Q-fuzzy left R-submodules of near-rings with respect to T-norms, Advances in Fuzzy Mathematics, **4(2)** (2009) 137–145.*

10. *A. Solairaju and R. Nagarajan, Q-fuzzy left R-subgroups of near-rings with respect to t-norms, Antarctica Journal of Mathematics, **5(1–2)** (2008) 59–63.*

11. *K. Vanathi, V.S.A. Subramanian and K. Arjunan, A study on anti Q-fuzzy subsemiring of a semiring, International Journal of Mathematical Sciences and Engineering Applications, **6**(2012) 345–352.*

12. *K. Vanathi, V.S.A. Subramanian and K. Arjunan, Notes on Q-intuitionistic fuzzy subsemiring of a semiring, International Journal of Mathematical Research, **3(2)**(2014) 15–24.*

13. *L.A. Zadeh, Fuzzy Sets, Information and Control **8** (1965) 338–353.*